

Government General Degree College, Dantan-II
Internal Assessment/4th Semester/Session : 2019-2020

Core-10

Subject-**Mathematics**(Honours)

Full marks-30

1. Answer **any six** questions: $6 \times 5 = 30$
2. The necessary and sufficient condition for a vector $\vec{r} = \vec{f}(t)$ to have a constant magnitude is $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$.
3. Prove that, $\text{curl curl } \vec{f} = \nabla(\nabla \cdot \vec{f}) - \nabla^2 \vec{f}$.
4. Evaluate $\iint_R (x^2 + y^2) dx dy$ over R bounded by $y=x^2$, $x=2$, $y=1$.
5. Find the equation of the tangent plane and the normal to the surface $xz^2 + x^2y = z$ at the point $(1, -3, 2)$.
6. Define irrotational and solenoidal vector. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ when $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.
7. If $\vec{a} = 3t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{b} = \sin t\hat{i} - 2\cos t\hat{j}$ then find $\frac{d}{dt}(\vec{a} \times \vec{b})$.
8. If $\vec{r} = (a \cos t)\hat{i} + (a \sin t)\hat{j} - (at \tan \alpha)\hat{k}$, then prove that $\left[\frac{d\vec{r}}{dx} \frac{d^2\vec{r}}{dt^2} \frac{d^3\vec{r}}{dt^3} \right] = a^3 \tan \alpha$
9. Find $\vec{t}, \vec{b}, \vec{n}$ for the circular helix $\vec{r} = a(\cos \theta, \sin \theta, \theta \cot \beta)$. Find also expression for curvature and torsion at a point on the curve.