

Government General Degree College, Dantan-II
Internal Assessment/4th Semester/Session : 2019-2020
Core-SEC2(Graph Theory)
Subject-**Mathematics**(Honours)
Full marks-20

Answer **any four** questions: $4 \times 5 = 20$

1. (i) Prove that the sum of the degrees of all vertices of a graph is an even integer.
(ii) Define complete graph with example.
2. (i) Prove that a simple graph with at least two vertices of same degree.
(ii) Does there exist a simple graph with five vertices having degree 2,2,4,4,4? Justify.
3. Prove that a simple graph with n vertices and m components can have at most $(n-m)(n-m+1)$ edges.
4. Define path and circuit. If a graph G contains exactly two vertices of odd degree then show that there exists a path between two vertices. Define cycle.
5. Define Euler circuit. If a connected graph G is Eulerian, then every vertex of G has even degree.
6. Define directed graph. Show that in a graph there are even numbers of vertices of odd degree.
7. Draw a graph having the given properties or explain why no such graph exists.
 - (a) Simple graph, four vertices each of degree two.
 - (b) Simple graph with five vertices having degree 3,3,3,3,4.
 - (c) Four edges; four vertices having degrees 1,2,3,4.
How many vertices are there in a graph with 15 edges if each vertex is of degree 3?
8. If G be a graph and u, v be two vertices of G such that $u \neq v$. If there is a trail from u to v then show that there is a path from u to v . Let G be a connected graph with at least two vertices. If the number of edges in G is less than the number of vertices, then prove that G has a vertex of degree one.