Answer **any nine** questions: $9 \times 5 = 45$

1.Show that $x_1 = 2$, $x_2 = 3$, $x_3 = 1$ is a feasible solution of the system of equations
$$2x_1 + x_2 + 4x_3 = 11,$$
$$3x_1 + x_2 + 5x_3 = 14.$$
Reduce the F.S. to B.F.S.

2.Make a graphical representation of the set of constraints of the following L.P.P. Find the corner points of the feasible region . Then solve the problem graphically :

Maximize, $z = 2x_1 + x_2$

Subject to,
$$x_1 + x_2 \geq 5,$$
$$2x_1 + 3x_2 \leq 20,$$
$$4x_1 + 3x_2 \leq 25,$$

$x_1, x_2 \geq 0$.

3.Use Charnes Big M-method (method of penalty) to

Maximize, $z = 5x_1 - 2x_2 + 3x_3$

Subject to,
$$2x_1 + 2x_2 - x_3 \geq 2,$$
$$3x_1 - 4x_2 \qquad \leq 3,$$
$$x_2 + 3x_3 \leq 5,$$

$x_1, x_2, x_3 \geq 0$.

4.Use duality to obtain an optimal solution , if any , to the L.P.P.

Maximize, $z = 5x_1 - 2x_2 + 3x_3$

Subject to,
$$2x_1 + 2x_2 - x_3 \geq 2,$$
$$3x_1 - 4x_2 \qquad \leq 3,$$
$$x_2 + 3x_3 \leq 5,$$

$x_1, x_2, x_3 \geq 0$.

5.Define convex combination and convex set. If x_1 , x_2 be real , show that set $X = \{(x_1, x_2) : 9x_1^2 + 4x_2^2 \leq 36\}$ is a convex set.

6.A businessman has the options of investing his money in two plans. Plane A, guarantees that each rupee invested will earn seventy paise a year hence while plan B guarantees that each rupees invested will earn two rupees two years hence. In plan B, only investment for periods that are multiple of two years are allowed. The problem is how should he invest ten thousand rupees in order to maximize the earning at the end of three years.

 Formulate this problem as a linear programming model.

7. Find two basic solutions of the following set of equations:

$$2x_1 + x_2 + 4x_3 = 11,$$
$$3x_1 + x_2 + 5x_3 = 14.$$

8.Define the convex combination and Hyperplane.

Show that A hyperplane is a convex set.

9. Show that the set $X = \{(x_1, x_2) \in \mathbb{R}^2 : 9x_1^2 + 4x_2^2 \leq 36 \}$ is convex set.

10. Show that the set

$X = \{(x_1, x_2) \in \mathbb{R}^2 : x_1 + x_2 \leq 50 , x_1 + 2x_2 \leq 80, 2x_1 + x_2 \geq 20, x_1, x_2 \geq 0\}$ is convex set.

11..Prove that dual of the dual is primal.

12. If x be any feasible solution to the primal problem and v be any feasible solution of the dual problem, then prove that $cx \leq b'v$.