Government General Degree College,Dantan-II

Internal Assessment/4th Semester/Session : 2019-2020

Core-10

Subject-**Mathematics**(Honours)

Full marks-30

Answer **any six** questions: $6 \times 5 = 30$

1.Define zero ring. Prove that if I be the unity in a ring then it is unique.

2.In a ring $(R, +, \cdot)$ prove that

$(i) a \cdot 0 = 0 \cdot a = 0$, for all $a \in R$, 0 being zero element in R,

$(ii)\ a \cdot (-b) = (-a) \cdot b = -(a \cdot b)$, for all $a, b \in R$,

$(iii)\ (-a) \cdot (-b) = a \cdot b$, for all $a, b \in R$.

3.Define divisors of zero in a ring.

Examine whether the ring of matrices $\left\{\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} : a, b \in \mathbb{R}\right\}$ contains divisors of zero.

4.Define unit in a ring.

Prove that if a be a unit in a ring with unity then a is not divisors of zero.

5.Define characteristic of a ring. Prove that the number of elements in a finite ring of characteristic 2 is 2^k for some positive integer k.

6. Prove that a ring containing 6 elements is commutative.

Define Nilpotent element and idempotent element in a ring.

7.Prove that the characteristic of an integral domain is either zero or prime.

8.Let U, W be two subspaces of a vector space V over a field F . Show that $U \cup W$ is a subspace of V if and only if either $U \subset W$ or $W \subset U$.

9.Prove that the set of vectors $\{(1,2,2), (2,1,2), (2,2,1)\}$ is linearly independent in $\mathbb{R}^3$.

10.State and prove extension theorem on independent set of vectors in a vector space.